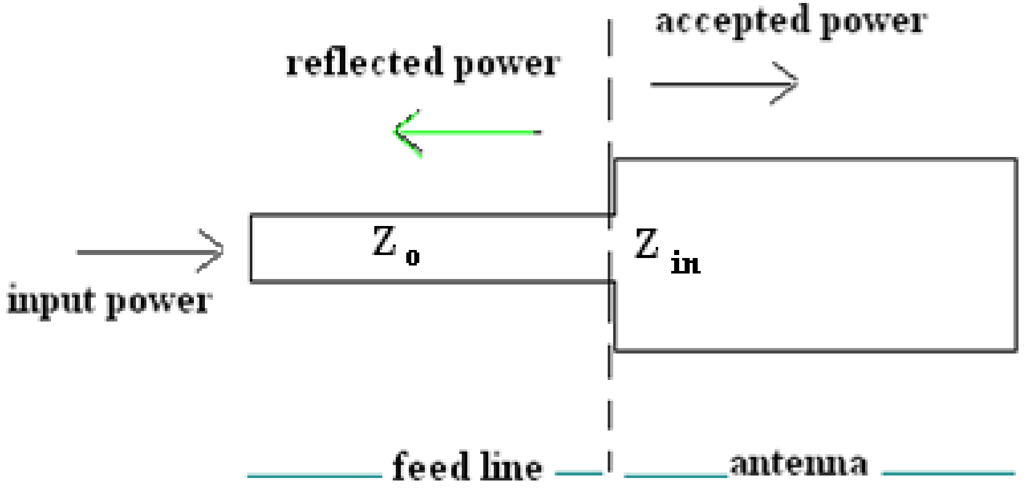
Antenna efficiency

Take into account losses at the input terminals and within the structure of the antenna.



– *Conduction and dielectric losses.*
– *Mismatch Reflections between the transmission line and the antenna.*

$$e_t = e_{cd} e_r$$

Where:

$e_t = {}^{P_{rad}}/_{P_{input}}$ Total efficiency

$e_{cd} = {}^{P_{rad}}/_{P_{accept}}$ Radiation efficiency(consider conduction and dielectric losses)

$e_r = {}^{P_{accept}}/_{P_{input}} = 1 - |\Gamma|^2$ Reflection efficiency (consider Mismatch loss)

$\boxed{\Gamma = \dfrac{Z_{in} - Z_0}{Z_{in} + Z_0},}$ $\boxed{\mathrm{VSWR} = \dfrac{1 + |\Gamma|}{1 - |\Gamma|}.}$

Γ = voltage reflection coefficient at the input terminals of the antenna

Z_{in} = antenna input impedance,

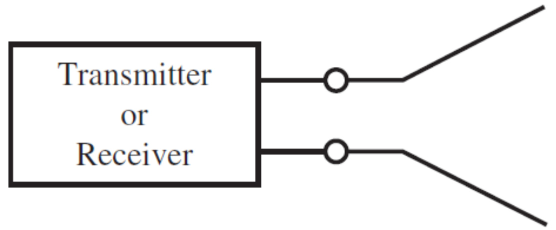
Z_0 = characteristic impedance of the transmission line

VSWR = voltage standing wave ratio

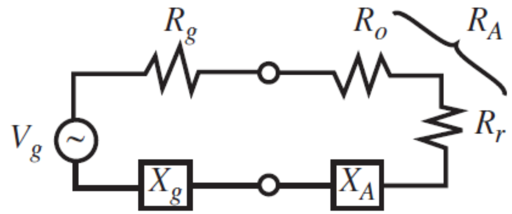
Efficiency is very close to: 100% (or 0 dB) for dish, horn antennas, or half-wavelength dipoles with no lossy materials around them, and 20%- 70% for Mobile antenna(microstrip) ,losses due to material surround antenna, and dielectric losses

Fundamental Parameters of Antennas

Radiation efficiency e_{cd}



(*a*) General antenna model.



(*b*) Equivalent model for a transmitting antenna.

RA represents dissipation (radiation + ohmic losses)

small antennas can have significant ohmic losses but other antennas usually have ohmic losses that are small compared to their radiation dissipation

$$e_{cd} = \frac{P_{rad}}{P_{accept}} = \frac{P_{rad}}{P_{rad} + P_o} = \frac{\frac{1}{2}R_r|I_A|^2}{\frac{1}{2}R_r|I_A|^2 + \frac{1}{2}R_o|I_A|^2} = \frac{R_r}{R_r + R_o} = \frac{R_r}{R_A}$$

where

P_{rad} = power radiated
P_o = power dissipated in ohmic losses on the antenna
$P_{accept} = P + P_o$ = power accepted by the antenna

radiation efficiency must be as close to 100% as possible in high_power transmitting antennas for two reasons:
the ohmic losses cost money in power consumption and the heat generated can possibly damage or destroy the antenna

Fundamental Parameters of Antennas

Example 1

A monopole antenna (h=0.787 m) used on cars for reception of AM and FM radio signals; assume monopole antenna has a good ground plane. The antenna is made of steel 1/8-in in diameter (a=0.15875 cm). For an operating frequency in the AM band of 1 MHz (λ=300 m) compute radiation efficiency of the antenna. .if this type of antenna (*short monopole*) has following relations for radiation resistance and loss resistance

$$R_r = 40\pi^2 \left(\frac{\Delta z}{\lambda}\right)^2 \Omega \qquad R_o = \frac{\Delta z}{2\pi a}\frac{R_s}{3}$$

Where $R_s = \sqrt{\frac{\omega\mu}{2\sigma}}$ $\qquad \sigma = 2*10^6$ S/m

Solution:

$$R_r = 40\pi^2 \left(\frac{0.787}{300}\right)^2 = 0.00271\ \Omega$$

$$R_s = \sqrt{\frac{2\pi \times 10^6 \cdot 4\pi \times 10^{-7}}{2 \cdot 2 \times 10^6}} = 1.405 \times 10^{-3}\ \Omega$$

$$R_o = \frac{h}{2\pi a}\frac{R_s}{3} = \frac{0.787}{2\pi \cdot 1.5875 \times 10^{-3}} \frac{1.40 \times 10^{-3}}{3} = 0.0370\ \Omega$$

radiation efficiency $\quad e_{cd} = \frac{R_r}{R_r + R_o} = \frac{0.00271}{0.00271 + 0.0370} = 6.82\%$

The low efficiency in this broadcast reception application is overcome by using a high-power transmitter operating into a tall, efficient antenna. Thus, the cost and complexity of the system are concentrated into the transmitting station, allowing for inexpensive and simple receiving antennas

Gain

- *Gain* of an antenna (in a given direction) is defined as "the ratio of the intensity, in a given direction, to the radiation intensity that would be obtained if the power accepted by the antenna were radiated isotropically.
- *Gain does not account for losses arising from* ***impedance mismatches***

$$\boxed{G(\theta, \phi) = e_{cd} D(\theta, \phi)}$$

the maximum value of the gain

$$\boxed{G_0 = G(\theta, \phi)|_{\max} = e_{cd} D(\theta, \phi)|_{\max} = e_{cd} D_0}$$

Absolute Gain

Take into account losses arising from impedance mismatches

$$\boxed{G_{abs} = e_r G(\theta,\phi) = (1 - |\Gamma|^2) G(\theta,\phi) = e_r e_{cd} D(\theta,\phi) = e_t D(\theta,\phi).}$$

- For the maximum values

$$\boxed{G_{0abs} = e_t D_0.}$$

$$G_0(\text{dB}) = 10 \log_{10}[e_{cd} D_0 \text{ (dimensionless)}]$$

Example 2

Example 2.10

A lossless resonant half-wavelength dipole antenna, with input impedance of 73 ohms, is connected to a transmission line whose characteristic impedance is 50 ohms. Assuming that the pattern of the antenna is given approximately by

$$U = B_0 \sin^3 \theta$$

find the maximum absolute gain of this antenna.

$$U|_{\max} = U_{\max} = B_0$$

$$P_{\rm rad} = \int_0^{2\pi} \int_0^{\pi} U(\theta, \phi) \sin\theta \, d\theta \, d\phi = 2\pi B_0 \int_0^{\pi} \sin^4 \theta \, d\theta = B_0 \left(\frac{3\pi^2}{4}\right)$$

$$D_0 = 4\pi \frac{U_{\max}}{P_{\rm rad}} = \frac{16}{3\pi} = 1.697$$

Since the antenna was stated to be lossless, then the radiation efficiency $e_{cd} = 1$.

Thus, the total maximum gain is equal to

$$G_0 = e_{cd} D_0 = 1(1.697) = 1.697$$

which is identical to the directivity because the antenna is lossless.

There is another loss factor which is not taken into account in the gain. That is the loss due to reflection or mismatch losses between the antenna (load) and the transmission line. This loss is accounted for by the reflection efficiency

$$e_{\rm t} = (1 - |\Gamma|^2) = \left(1 - \left|\frac{73 - 50}{73 + 50}\right|^2\right) = 0.965$$

$$e_{\rm t} = e_r e_{cd} = 0.965$$

$$G_{0abs} = e_{\rm t} D_0 = 0.965(1.697) = 1.6376$$

Bandwidth

defined as the range of frequencies where performance of antenna(antenna characteristic as input impedance ,pattern, polarization ,gain, radiation efficiency,…) conforms to specific standards (according to antenna application).

For narrowband antennas, the BW is expressed as a percentage of the frequency difference over the center frequency:

$$\boxed{BW = \frac{f_{upper} - f_{lower}}{f_0} \cdot 100\ \%.}$$

Usually, $f_0 = (f_{upper} + f_{lower}) / 2$

For broadband antennas $\boxed{BW = f_{upper} / f_{lower}}$

The Bandwidth for Several Common Antennas.(antenna-theory.com)

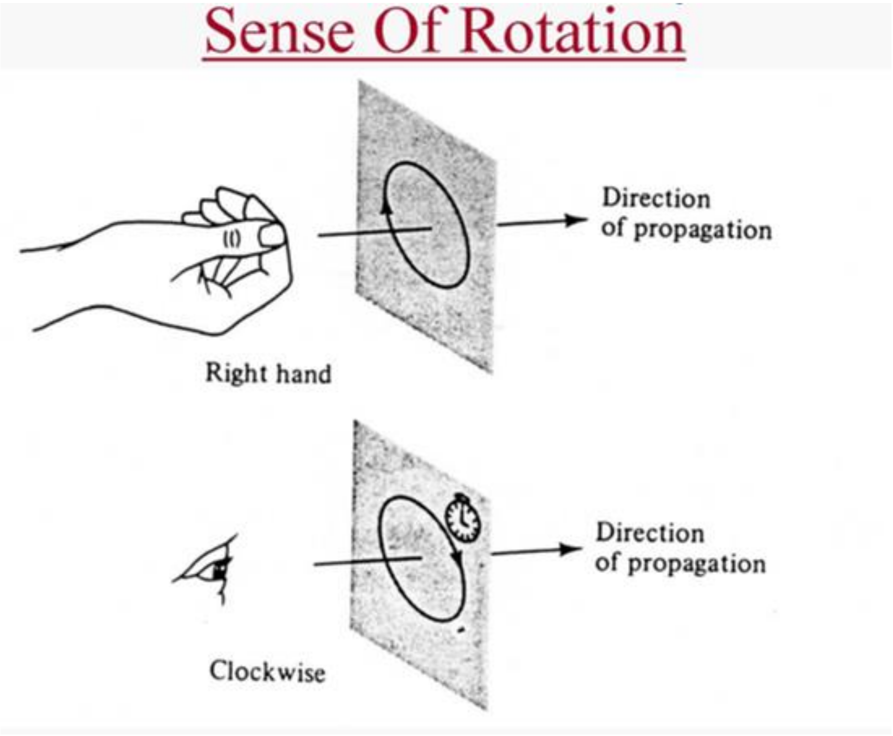
Antenna	Center frequency	Frequency range	Fractional BW fU-fL/fo	Ratio fU/fL	Percentage BW
Narrow band antenna					
Patch	1000 MHz	985-1015 MHz	0.03	1.0305:1	3%
Dipole	1000 MHz	960-1040 MHz	0.08	1.083:1	8%
Broad band antenna					
Horn	1000 MHz	154-1848 MHz	1.694	12:1	169.4%
Spiral	1000 MHz	95-1900 MHz	1.805	20:1	180.5%

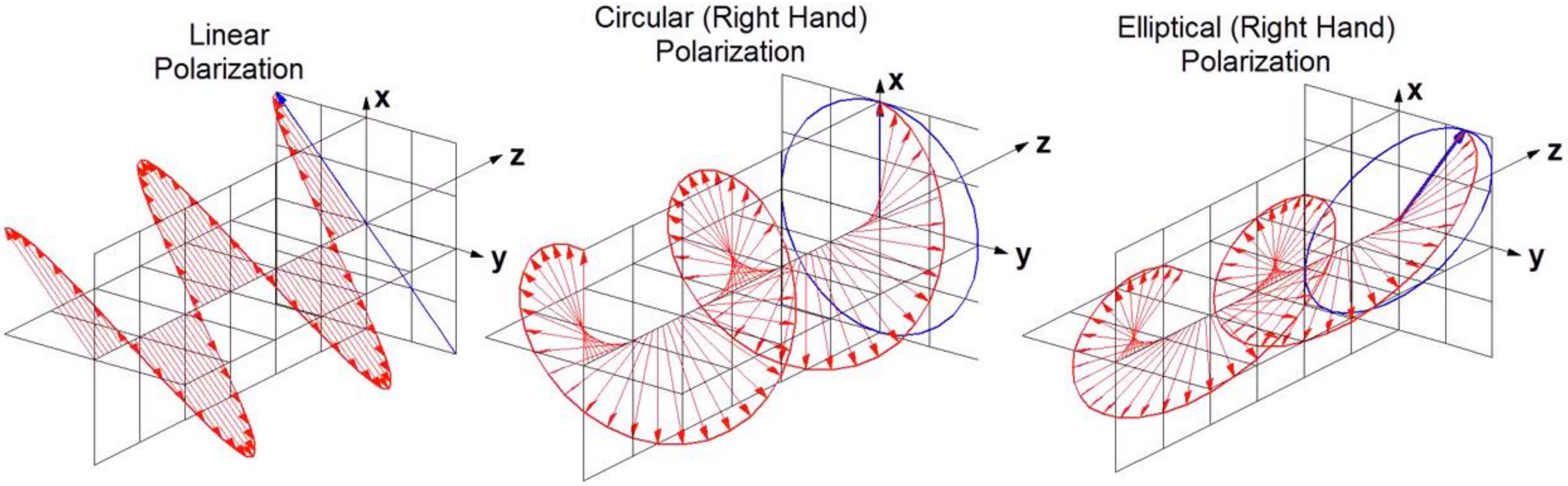
- <u>Polarization of EM fields</u>

The polarization of the EM field describes the orientation of its vectors at a given point and how it varies with time. In other words, it describes the way the direction and magnitude of the field vectors (usually **E**) change in time.

- **three types of polarization exist for harmonic fields:**

linear, circular and elliptical.




Linear
Polarization
Circular (Right Hand)
Polarization
Elliptical (Right Hand)
Polarization
x
z
y
x
z
y
x
z
y

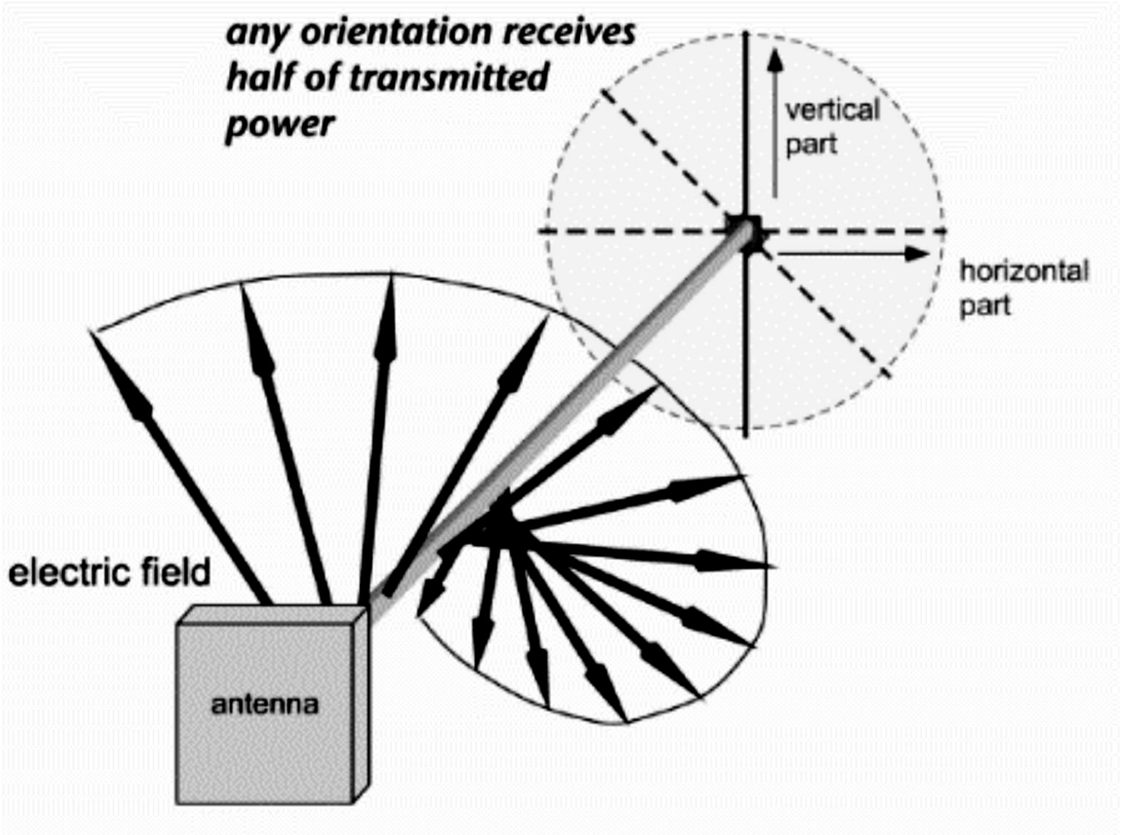
Examples of Polarization

- AM Radio (0.540-1.6 MHz): linearly polarized (vertical) (Note 1)
- FM Radio (88-108 MHz): linearly polarized (horizontal)
- TV (VHF, 54-216 MHz; UHF, 470-698 MHz: linearly polarized (horizontal) (some transmitters are CP)
- Cell phone antenna (about 2 GHz, depending on system): linearly polarized (direction arbitrary)
- DBS Satellite TV (11.7-12.5 GHz): transmits both LHCP and RHCP (frequency reuse) (Note 2)
- GPS (1.574 GHz): RHCP (Note 2)

Notes:

1) Low-frequency waves travel better along the earth when they are polarized vertically instead of horizontally.
2) Satellite transmission often uses CP because of rotation of waves in the ionosphere

waves bounce of the ground and other objects so do not maintain their original orientation anyway. so alignment of linearly polarized sending and receiving antennas is more difficult to achieve.
These difficulties are somewhat circumvented by circular polarization of waves.



circular polarization has a number of benefits for areas such as satellite applications where it helps overcome the effects of propagation anomalies and ground reflections

$$\vec{E}(z,t) = E_{x0}\cos(\omega t + kZ + \varphi_x)\,\hat{a}_x + E_{y0}\cos(\omega t + kZ + \varphi_y)\;\hat{a}_y$$

Linear polarized

A. $E_{x0} \neq 0$, $E_{y0} = 0$

B. E_{x0}=0 , $E_{y0} \neq 0$

C. $E_{x0} \neq 0$, $E_{y0} \neq 0$

$$\varphi_y\text{-}\varphi_x = \pm n\pi \qquad n = 0,1,\ldots..$$

Circular polarized:

$$E_{x0} = E_{y0}$$

$$\varphi_y\text{-}\varphi_x = \begin{cases} -\left(\frac{\pi}{2} + n\pi\right) & \text{for CCW(LH)} \\ \frac{\pi}{2} + n\pi & \text{for CW(RH)} \end{cases} \qquad n = 0,1,2,\ldots$$

Elliptical polarized:

- $E_{x0} \neq E_{y0}$

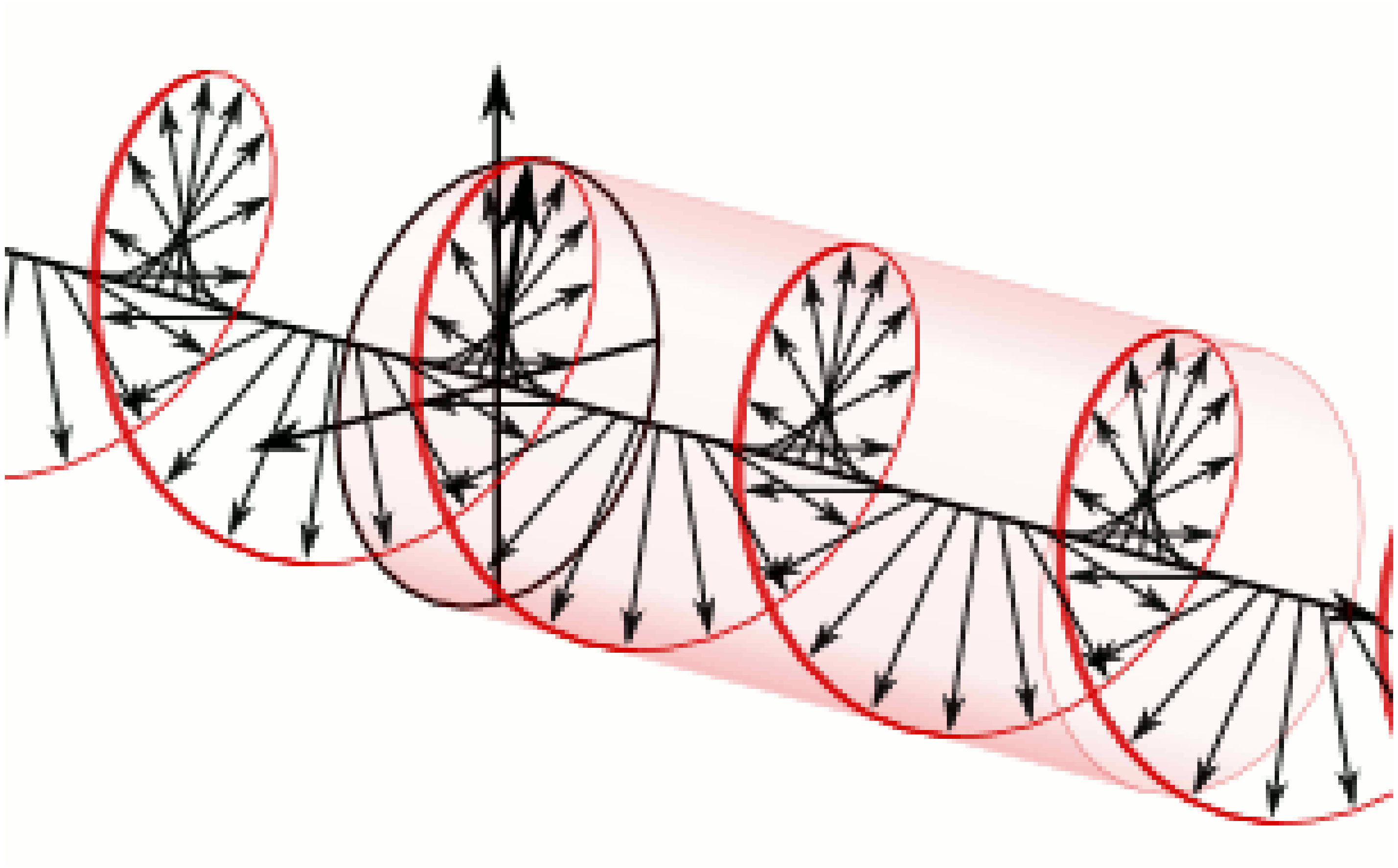
$$\varphi_y\text{-}\varphi_x = \begin{cases} -\left(\frac{\pi}{2} + n\pi\right) & \text{for CCW(LH)} \\ \frac{\pi}{2} + n\pi & \text{for CW(RH)} \end{cases} \qquad n = 0,1,2,\ldots$$

- $E_{x0} = E_{y0}\ and$ $\quad \varphi_y - \varphi_x \neq \pm\frac{\pi}{2}$ not circular
- $\varphi_y - \varphi_x \neq 0 \text{ or } \pi$ not linear

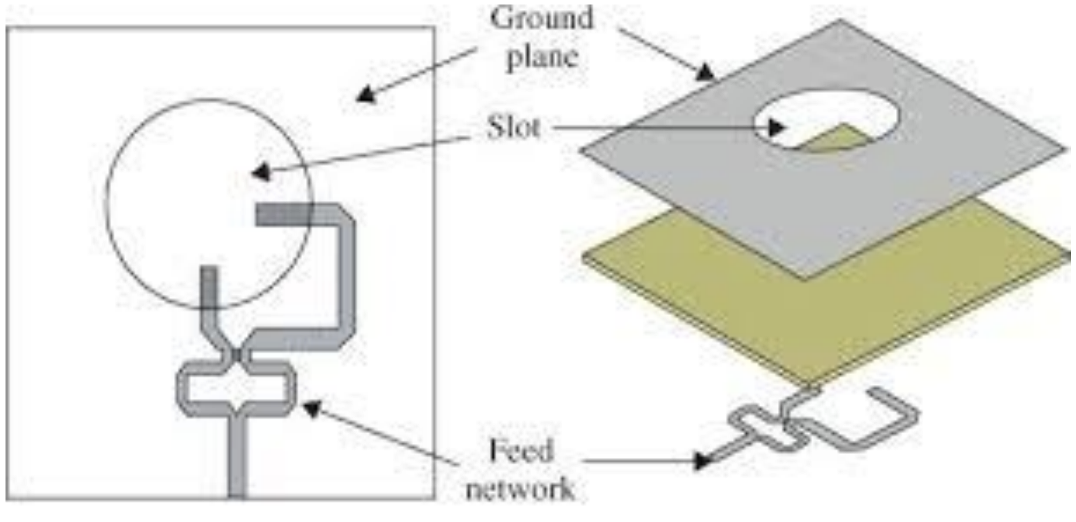
When the ellipse is aligned with the Type equation here.principal axes , the major (minor) axis is equal to *Exo(Eyo)* or *Eyo(Exo)* and the axial ratio is equal to *Exo/Eyo* or *Eyo/Exo*.

If the direction of wave propagation is reversed (i.e., $+z$ direction), the phases in (2-60) and (2-61) for CW and CCW rotation must be interchanged.

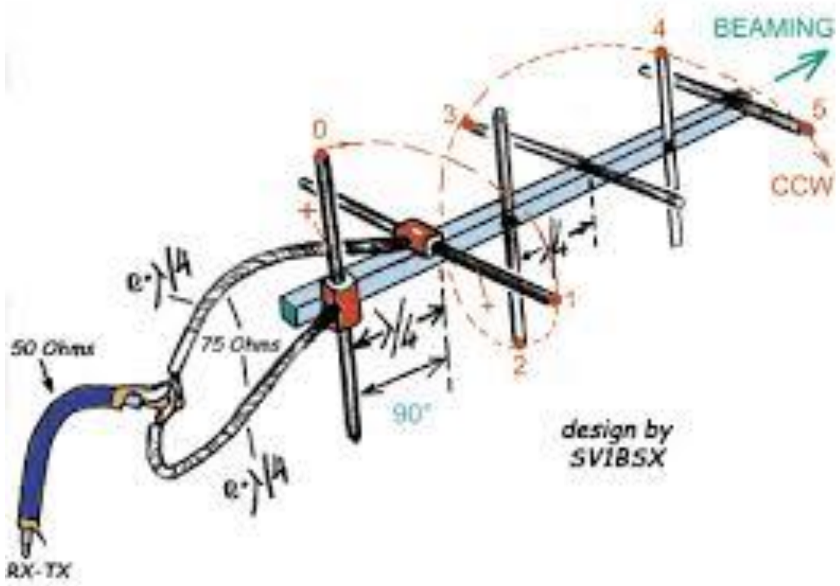
- A left-handed/anti-clockwise circularly polarized wave as defined from the point of view of the source. It would be considered right-handed/clockwise circularly polarized if defined from the point of view of the receiver

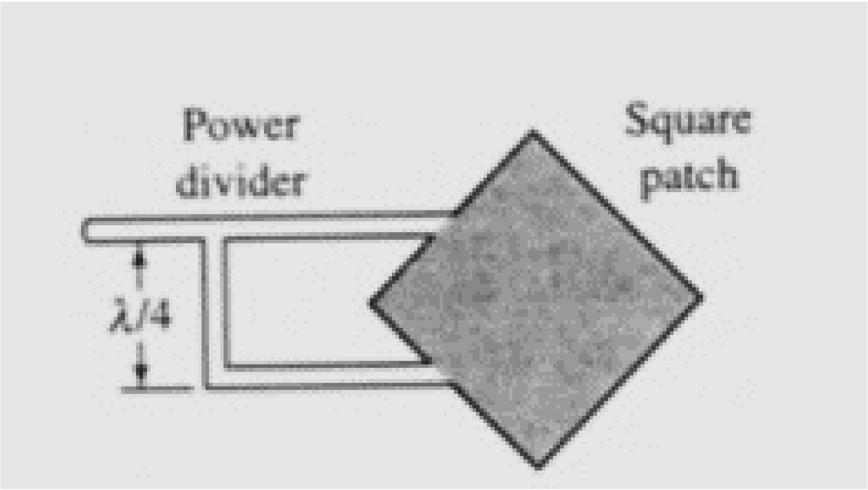


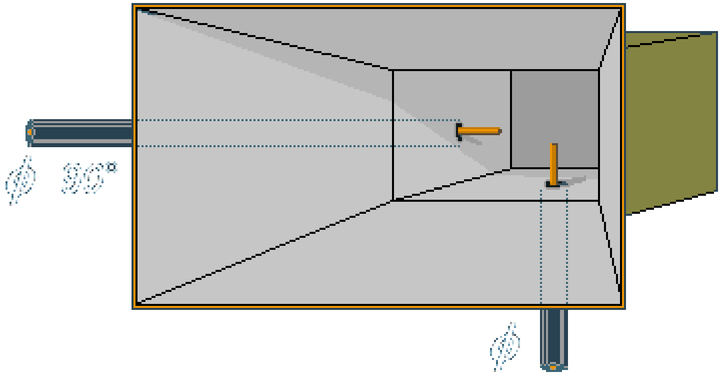
Examples for CP antennas



(a) Top view (b) 3-D view







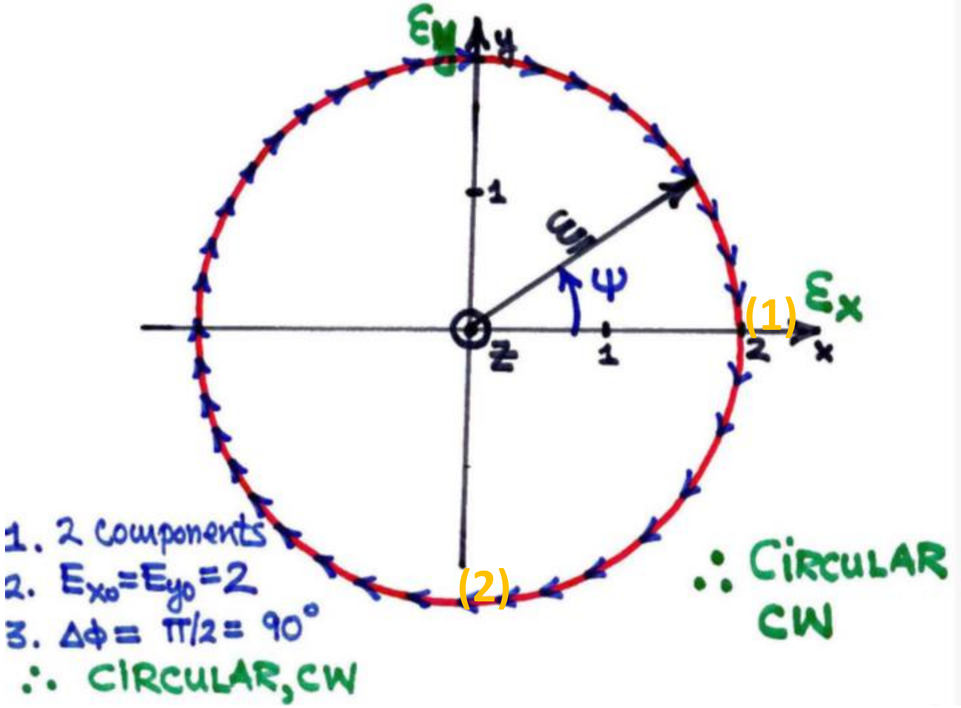
Example 3

$$\underline{E} = \left(2\hat{a}_x + j2\hat{a}_y\right)e^{jkz} = \left(2\hat{a}_x + 2\hat{a}_y e^{j\pi/2}\right)e^{+jkz}$$

Solution:

$$\vec{E}(z,t) = E_{x0}\cos(\omega t + kZ + \varphi_x)\,\hat{a}_x + E_{y0}\cos(\omega t + kZ + \varphi_y)\,\hat{a}_y$$

Since e^{jkz} Propagation in - $\hat{a}_z$ I am looking to direction of propagation, $E_{x0} = E_{y0}$=2 , $\emptyset_y$-$\emptyset_x$=90



$at\ Z = 0$ $E_x = 2\cos(\omega t)$, $E_y = 2\cos(\omega t$+90)=-sin(ωt),

at (1) ωt =0 E_x=2 , E_y=0

at (2) ωt =90 E_x=0 , E_y=-2

Example 4

- A wave traveling normally outward from the page (toward the reader) direction resultant of two elliptically polarized waves, one with components
- of E given by:

$$\mathscr{E}'_y = 3\cos\omega t$$

$$\mathscr{E}'_x = 7\cos\left(\omega t + \frac{\pi}{2}\right)$$

And the other with components given by:

$$\mathscr{E}''_y = 2\cos\omega t$$

$$\mathscr{E}''_x = 3\cos\left(\omega t - \frac{\pi}{2}\right)$$

(a) What is the axial ratio of the resultant wave?

(b) Does the resultant vector E rotate clockwise or counterclockwise?(**from your view**)

(a) $E_x = E'_x + E''_x = 7\cos(\omega t + \frac{\pi}{2}) + 3\cos(\omega t - \frac{\pi}{2})$

$= -7\sin\omega t + 3\sin\omega t = -4\sin\omega t$

$= 4\cos(\omega t + \frac{\pi}{2})$

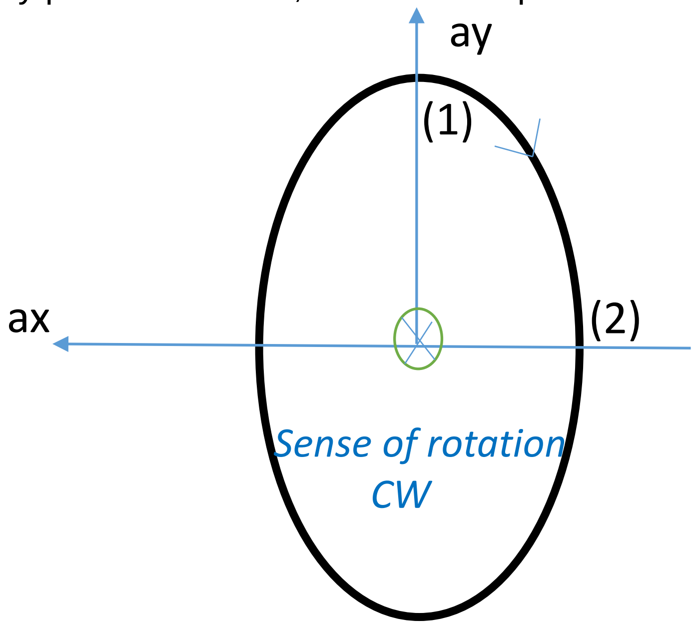
$E_y = E'_y + E''_y = 3\cos\omega t + 2\cos\omega t = 5\cos\omega t$

$\varphi_y - \varphi_x = -\frac{\pi}{2} \longrightarrow$ Rotation in CCW

$AR = \frac{5}{4} = 1.25$

(b) At $\omega t = 0$, $\vec{E} = 5\hat{a}_y$ (1)

At $\omega t = \pi/2 \Rightarrow \vec{E} = -4\hat{a}_x \Rightarrow$ Rotation in CCW



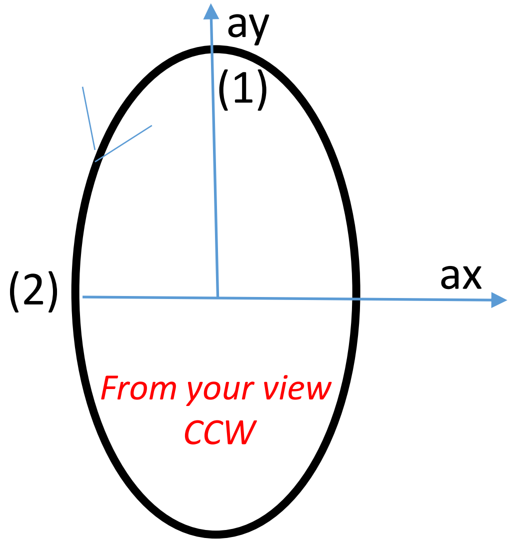
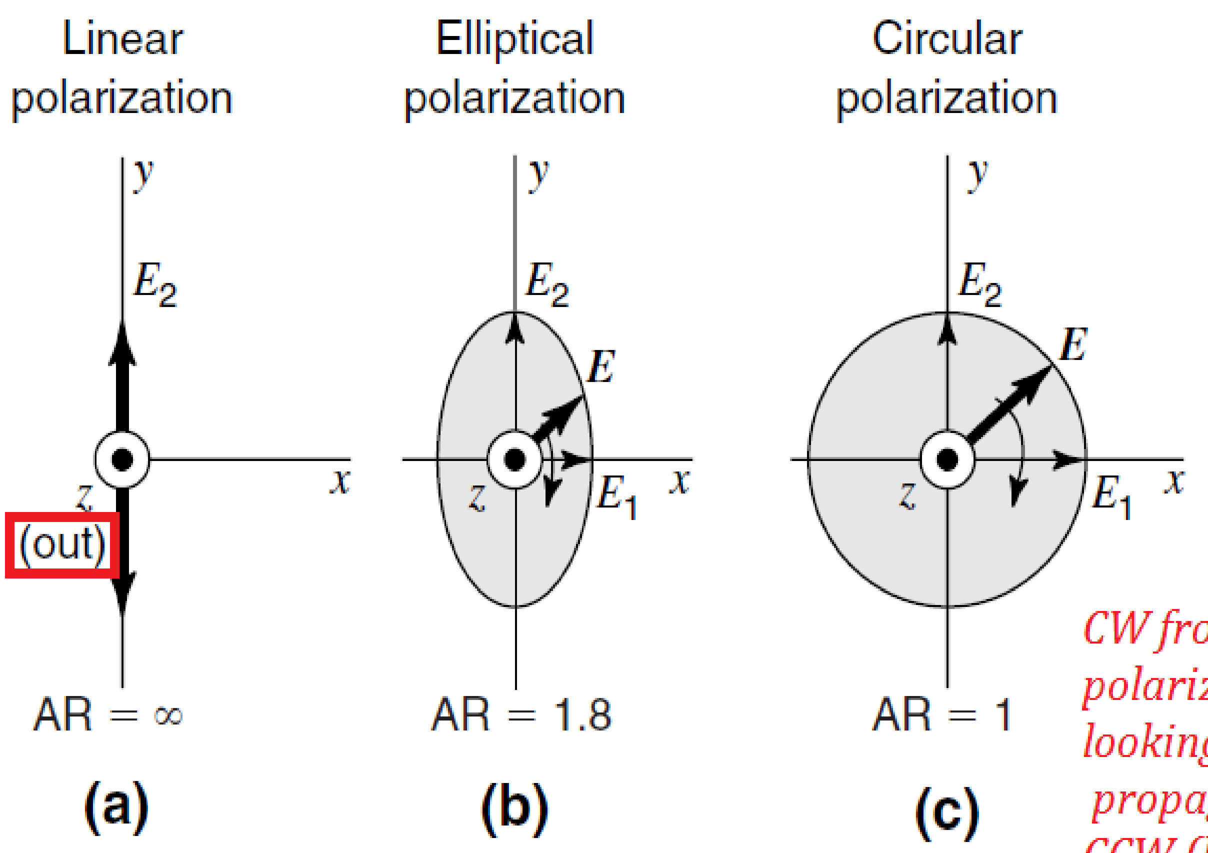


Figure 2–21
(*a*) Linear, (*b*) elliptical, and (*c*) circular polarization for left-circularly polarized wave approaching.



CW from your view polarizing sense looking to wave propagation
CCW (left)

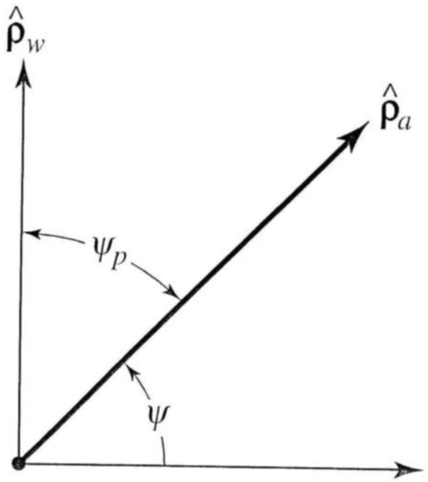
Direction of propagation +az

Polarization Loss Factor

$$PLF = |\hat{\rho}_w . \hat{\rho}_a|^2 = \left|cos\psi_p\right|^2$$

$$0 \leq PLF \leq 1$$

$$PLF(dB) = 10\log_{10}|\hat{\rho}_w . \hat{\rho}_a|^2$$



Cross polarization:

Every antenna radiates in a desire polarization as it was designed to, but it has a leakage that radiates in the perpendicular polarization to the desired one. The ratio between undesired polarization to the desired polarization is the cross polarization.

Antennas are designed to have as low cross polarization as possible

Desired polarization	Undesired perpendicular polarization
Vertical	Horizontal
Horizontal	Vertical
RHCP	LHCP
LHCP	RHCP

Example 5

Example 2.11:

Find PLF

$$\underline{E}_w^i = \hat{a}_x E_0(x,y) e^{-jkz}$$

$$\underline{E}_a \simeq (\hat{a}_x + \hat{a}_y) E(r,\theta,\phi)$$

Solution:

$$\underline{E}_a = (\hat{a}_x + \hat{a}_y) E(r,\theta,\phi) = \left(\frac{\hat{a}_x + \hat{a}_y}{\sqrt{2}}\right)\sqrt{2} E(r,\theta,\phi)$$

$$\hat{\rho}_a = \frac{\hat{a}_x + \hat{a}_y}{\sqrt{2}}, \qquad \hat{\rho}_w = \hat{a}_x$$

$$PLF = \left|\hat{\rho}_w \cdot \hat{\rho}_a\right|^2 = \left|\hat{a}_x \cdot \left(\frac{\hat{a}_x + \hat{a}_y}{\sqrt{2}}\right)\right|^2 = \frac{1}{2}$$

$$PLF = \frac{1}{2} = 10\log_{10}\left(\frac{1}{2}\right) = -3 \text{ dB}$$

Example 6

Example 2.12:

$$\underline{E}_a = (\hat{a}_\theta - j\hat{a}_\phi)E(r,\theta,\phi)$$

Solution:

$$\underline{E}_a = (\hat{a}_\theta - j\hat{a}_\phi)E(r,\theta,\phi)$$

$$= \frac{(\hat{a}_\theta - j\hat{a}_\phi)}{\sqrt{2}}\sqrt{2}E(r,\theta,\phi)$$

$$\underline{E}_a = \hat{\rho}_a\sqrt{2}E(r,\theta,\phi), \quad \hat{\rho}_a = \frac{\hat{a}_\theta - j\hat{a}_\phi}{\sqrt{2}}$$

CW-CW (Maximum)

Antenna transmit wave to +z direction

$$\text{CW:} \quad \hat{\rho}_a = \left(\frac{\hat{a}_\theta - j\hat{a}_\phi}{\sqrt{2}} \right), \quad \text{CW:} \quad \hat{\rho}_w = \left(\frac{\hat{a}_\theta + j\hat{a}_\phi}{\sqrt{2}} \right)$$

$$PLF = \left| \hat{\rho}_a \cdot \hat{\rho}_w \right|^2 = \left| \left(\frac{\hat{a}_\theta - j\hat{a}_\phi}{\sqrt{2}} \right) \left(\frac{\hat{a}_\theta + j\hat{a}_\phi}{\sqrt{2}} \right) \right|^2$$

$$PLF = \left| \frac{1+1}{2} \right|^2 = 1 = 0 \ \text{dB}$$

PLF for Transmitting and Receiving Linear Antennas

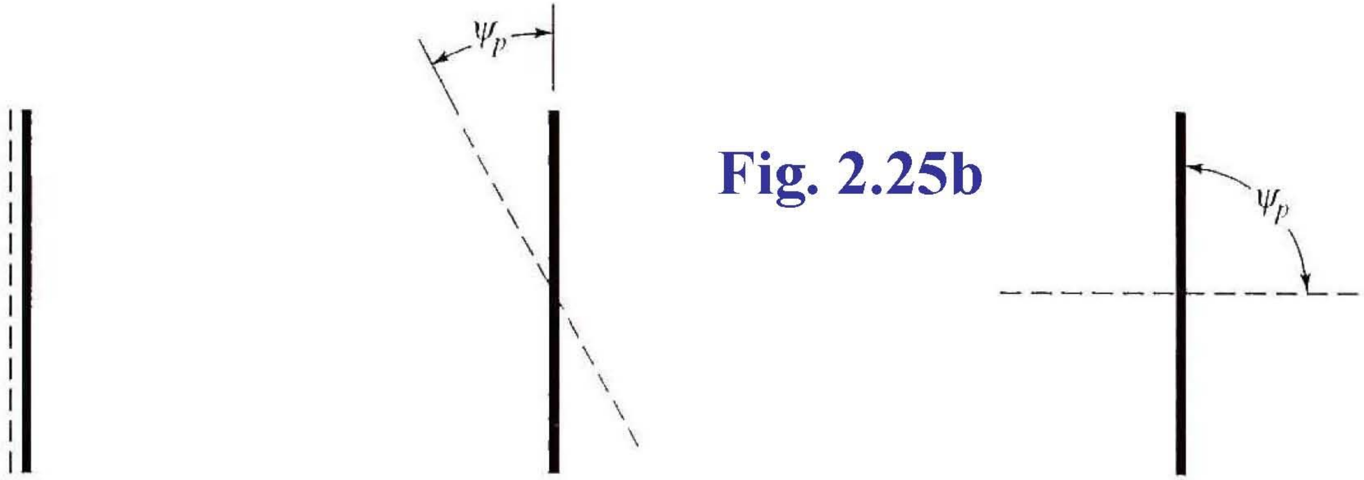


Fig. 2.25b

$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = 1$ (aligned)

$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = \cos^2 \psi_p$ (rotated)

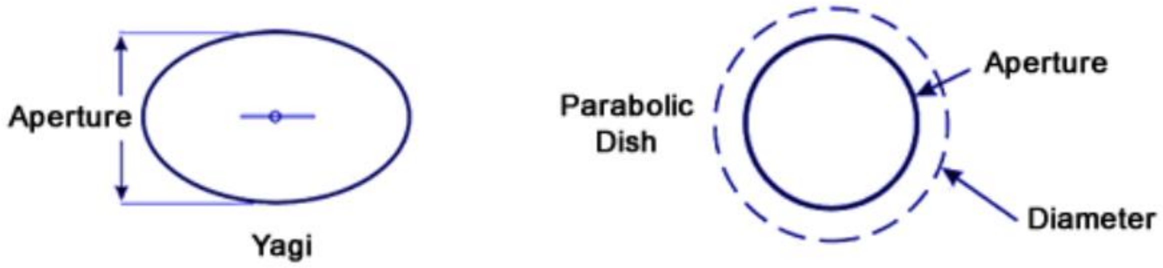
$\text{PLF} = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = 0$ (orthogonal)

Effective Area

the effective area simply represents how much power is captured from the plane wave and delivered by the antenna.

effective area is defined as “the ratio of the available power at the terminals of a receiving antenna to the power flux density of a plane wave incident on the antenna from that direction, the wave being polarization-matched to the antenna.

$$A_e = \frac{P_r}{W_{avg}}$$

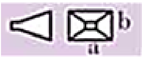


P_r is power delivered to the load

General formula to compute (no losses) :

$$A_e = \frac{\lambda^2}{4\pi} D_0 = \frac{\lambda^2}{\Omega}$$

Some Antenna Structures, Directivity and Effective Area

Antenna	*Dimension*	*Directivity*	*Effective area(Ae) compared to physical area(Ap)*
Half wave Dipole		$D=1.6$	$Ae=0.13*\lambda^2$
Horn Antenna		$D=10ab/\lambda^2$	$Ae=0.8\,Ap$
parabolic Reflector		$D=7*\pi\, r^2/\lambda^2$	$Ae=0.6\,Ap$

More General: Taking Into Account Losses

$$A_e = \frac{\lambda^2}{4\pi} \underbrace{e_o D(\theta,\phi)}_{G_{abs}(\theta,\phi)} \left|\hat{\rho}_w \cdot \hat{\rho}_a\right|^2$$

$$A_{em} = \frac{\lambda^2}{4\pi} \underbrace{e_o D_0}_{G_{0abs}} \left|\hat{\rho}_w \cdot \hat{\rho}_a\right|^2$$

$$\boxed{A_{em} = \boxed{\frac{\lambda^2}{4\pi} D_0} \underbrace{e_{cd}\left(1-|\Gamma|^2\right)}_{e_o} \underbrace{\left|\hat{\rho}_w \cdot \hat{\rho}_a\right|^2}_{e_p = PLF}} \quad (2\text{-}112)$$

Friis Transmission Equation

For transmitting antenna

$$D_t = \frac{U_t}{Prad/4\pi} = \frac{R^2 W_{tavg}}{Prad/4\pi}$$

$$W_{tavg} = \frac{D_t . Prad}{4\pi . R^2}$$

$$e_t = \frac{Prad}{P_{tinput}} \qquad W_{tavg} = \frac{D_t . e_t . P_{tinput}}{4\pi . R^2} = \frac{G_t . P_{tinput}}{4\pi . R^2}$$

At the receiver end:

$$A_{er} = \frac{\lambda^2}{4\pi} G_r . |\hat{\rho}_w . \hat{\rho}_a|^2 = \frac{P_r}{W_{tavg}}$$

$$P_r = \frac{G_t . P_{tinput}}{4\pi . R^2} \frac{\lambda^2}{4\pi} G_r . |\hat{\rho}_w . \hat{\rho}_a|^2$$

$$\boxed{\frac{P_r}{P_{tinput}} = \left(\frac{\lambda}{4\pi R}\right)^2 G_t \, G_r |\hat{\rho}_w . \hat{\rho}_a|^2}$$

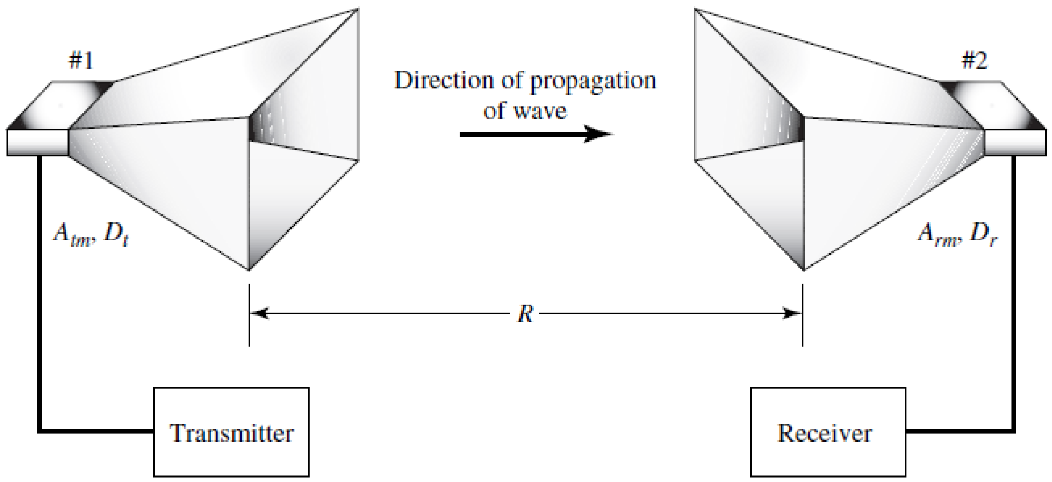


Figure 2.30 Two antennas separated by a distance R.

Example 7

Example 2.16

Two *lossless* *X*-band (8.2–12.4 GHz) horn antennas are separated by a distance of 100λ. The reflection coefficients at the terminals of the transmitting and receiving antennas are 0.1 and 0.2, respectively. The maximum directivities of the transmitting and receiving antennas (over isotropic) are 16 dB and 20 dB, respectively. Assuming that the input power in the lossless transmission line connected to the transmitting antenna is 2W, and the antennas are aligned for maximum radiation between them and are polarization-matched, find the power delivered to the load of the receiver.

Solution: For this problem

$$e_{cdt} = e_{cdr} = 1 \text{ because antennas are lossless.}$$

$$|\hat{\rho}_t \cdot \hat{\rho}_r|^2 = 1 \text{ because antennas are polarization-matched}$$

$$\left.\begin{aligned} D_t &= D_{0t} \\ D_r &= D_{0r} \end{aligned}\right\} \begin{array}{l} \text{because antennas are aligned for} \\ \text{maximum radiation between them} \end{array}$$

$$D_{0t} = 16 \text{ dB} \Rightarrow 39.81 \text{ (dimensionless)}$$

$$D_{0r} = 20 \text{ dB} \Rightarrow 100 \text{ (dimensionless)}$$

Using (2-118), we can write

$$\begin{aligned} P_r &= [1 - (0.1)^2][1 - (0.2)^2][\lambda/4\pi(100\lambda)]^2(39.81)(100)(2) \\ &= 4.777 \text{ mW} \end{aligned}$$

Example 8

2.85. Transmitting and receiving antennas operating at 1 GHz with gains (over isotropic) of 20 and 15 dB, respectively, are separated by a distance of 1 km. Find the maximum power delivered to the load when the input power is 150 W. Assume that the

(a) antennas are polarization-matched

$$\frac{P_r}{P_t} = |\hat{\rho}_t \cdot \hat{\rho}_r|^2 \left(\frac{\lambda}{4\pi R}\right)^2 G_{ot}\, G_{or}$$

$G_{ot} = 20\,dB \Rightarrow G_{ot}$ (power ratio) $= 10^2 = 100$

$G_{or} = 15\,dB \Rightarrow G_{or}$ (power ratio) $= 10^{1.5} = 31.623$

$f = 1\,GHz \Rightarrow \lambda = 0.3$ meters

$R = 1 \times 10^3$ meters

For $|\hat{\rho}_t \cdot \hat{\rho}_r|^2 = 1$

$$P_r = \left(\frac{0.3}{4\pi \times 10^3}\right)^2 (100)(31.623)(150 \times 10^{-3}) = 270.344\ \mu\ \text{Watts}$$

Example 9

2.72. For an *X*-band (8.2–12.4 GHz) rectangular horn, with aperture dimensions of 5.5 cm and 7.4 cm, find its maximum effective aperture (*in* cm^2) when its gain (over isotropic) is

(a) 14.8 dB at 8.2 GHz

(b) 16.5 dB at 10.3 GHz

(c) 18.0 dB at 12.4 GHz

$$A_{em} = \frac{\lambda^2}{4\pi} e_t D_o = \frac{\lambda^2}{4\pi} G_o$$

a. $G_o = 14.8$ dB $\Rightarrow G_o$ (power ratio) $= 10^{1.48} = 30.2$

$f = 8.2$ GHz $\Rightarrow \lambda = 3.6585$ cm

$$A_{em} = \frac{(3.6585)^2}{4\pi}(30.2) = 32.167 \text{ cm}^2$$

The physical aperture is equal to $A_p = 5.5(7.4) = 40.7 \text{ cm}^2$

b. $G_o = 16.5$ dB $\Rightarrow G_o$ (power ratio) $= 10^{1.65} = 44.668$

$f = 10.3$ GHz $\Rightarrow \lambda = 2.912$ cm

$$A_{em} = \frac{(2.912)^2}{4\pi}(44.668) = 30.142 \text{ cm}^2$$

c. $G_o = 18.0$ dB $\Rightarrow G_o$ (power ratio) $= 10^{1.8} = 63.096$

$f = 12.4$ GHz $\Rightarrow \lambda = 2.419$ cm

$$A_{em} = \frac{(2.419)^2}{4\pi}(63.096) = 29.389 \text{ cm}^2$$

Example 10

2.82. A lossless ($e_{cd} = 1$) antenna is operating at 100 MHz and its maximum effective aperture is 0.7162 m^2 at this frequency. The input impedance of this antenna is 75 ohms, and it is attached to a 50-ohm transmission line. Find the directivity (dimensionless) of this antenna if it is polarization-matched.

$$A_{em} = 0.7162 \ m^2$$

$$A_{em} = \frac{\lambda^2}{4\pi} \cdot \overset{1}{e_{cd}} (1 - |\Gamma|^2) \overset{1}{|\hat{\rho}_w \cdot \hat{\rho}_a|} \cdot D_0$$

Note that Max effective aperture consider losses

$$D_0 = \frac{A_{em}}{\frac{\lambda^2}{4\pi}(1-|\Gamma|^2)} \quad , \Gamma = \frac{75-50}{75+50} = 0.2 \quad , \lambda = \frac{3\times10^8}{100\times10^6} = 3m$$

$$D_0 = \frac{0.7162}{\frac{3^2}{4\pi}(1-|0.2|^2)}$$

$$D_0 = 1.0417$$